

Costs functions

Concept of Cost

- Cost, a key concept in economics, is the monetary expenses incurred by organizations for various purposes such as, acquiring resources, producing goods and service, advertising, and hiring workers.
- In other words, cost can be defined as monetary expenses that are incurred by organization for a specified thing or activity.

Kinds of costs

- Broadly there are two types of cost :
 1. Accounting Cost
 2. Economic Cost

Types of Costs

Accounting Costs

Opportunity cost
and Actual cost

Explicit cost and
Implicit Costs

Economic Costs

Fixed and Variable
Costs

Total, Average and
Marginal Cost

Incremental and
sunk costs

Accounting Costs

- **Accounting cost** is an explicit payment (that is, money changing hands) incurred by a firm.
- These are the expenses of an organization incurred during production and are entered in the book of accounts of the organization. Such as, wages, interest rent, cost of raw material, depreciation of capital goods and so on.
- **Accounting Profits = Total Revenue - Explicit Cost**

Opportunity and Actual Costs

- Cost is considered as the value of inputs, such as, land, labor and capital used for the production of goods and services. If an organization utilize an input to produce a particular good then the same input would not be available to produce another good.
- The cost incurred on the next best alternative that is forgone to acquire or produce a particular good is known as opportunity cost.

Actual cost

- Actual costs are those costs which are incurred by the organization on actual goods to carry out the production activities. These costs are incurred on purchasing raw materials, plant, machinery and other physical assets. Actual costs are the payments that are made in monetary terms and are recorded in the books of accounts.

Explicit and Implicit Costs

- **Explicit costs** A direct payment made to others in the course of running a business, such as wages, rent, and materials.
- **Implicit costs** is the opportunity **cost** equal to what a firm must give up in order to use a factor of production for which it already owns and thus does not pay rent. It is the opposite of an explicit **cost**, which is borne directly. This is true whether the costs are implicit or explicit. Both matter for firms' decisions.

Explicit vs. Implicit Costs: An Example

You need \$100,000 to start your business.

The interest rate is 5%.

- Case 1: borrow \$100,000
 - explicit cost = \$5000 interest on loan
- Case 2: use \$40,000 of your savings, borrow the other \$60,000
 - explicit cost = \$3000 (5%) interest on the loan
 - implicit cost = \$2000 (5%) *foregone* interest you could have earned on your \$40,000.
 - ***In both cases, total (exp + imp) costs are \$5000.***

Meaning of Economic Costs

- The economic cost is based on the cost of the alternative chosen and the benefit that the best alternative would have provided if chosen.
- Economic Profit = accounting cost + opportunity cost.

An example of economic cost would be the cost of attending college. The accounting cost includes all charges such as tuition, books, food, housing, and other expenditures.

The opportunity cost includes the salary or wage the individual could be earning if he was employed during his college years instead of being in school. So, the economic cost of college is the accounting cost plus the opportunity cost.

Economic Profit vs. Accounting Profit

- Economic profit is the monetary costs and opportunity costs a firm pays and the revenue a firm receives.

Economic profit = total revenue - (explicit costs + implicit costs).

- Accounting profit is the monetary costs a firm pays out and the revenue a firm receives. It is the bookkeeping profit, and it is higher than economic profit.

Accounting profit = total monetary revenue - total costs.

Fixed, variable and total cost

- **Fixed costs**
 - do not vary with changes in output
- **Variable costs**
 - vary with changes in output
- **Total costs**
 - the sum of fixed and variable costs at each level of output

Average Fixed and Average Variable costs

- **Average fixed costs (AFC)** are found by $AFC = \text{total fixed costs} / \text{output}$.

As fixed cost is divided by an increasing output, average fixed costs will continue to fall.

- **Average variable costs (AVC)** are found by :

$AVC = \text{total variable costs} / \text{output}$.

The *average variable cost (AVC)* curve will at first slope down from left to right, then reach a minimum point, and rise again.

Average total cost (ATC)

- **Average total cost (ATC)** can be found by adding : average fixed costs (AFC) + average variable costs (AVC).

OR

$$ATC = \text{total costs} / \text{output}$$

The ATC curve is also 'U' shaped because it takes its shape from the AVC curve

Marginal Costs

- Marginal Cost (MC)
 - the extra, or additional cost of producing one more unit of output

$$\text{Marginal Cost} = \frac{\text{Change in Total Costs}}{\text{Change in Quantity}}$$

Incremental costs and sunk costs

- Incremental costs are those costs that are incurred during the expansion of an organization. There are the added costs that are involved in changing the level of production or the nature of business activity. Expansion can be in form of man, machinery, material and so on.
- For instance : If firm purchases machinery the following incremental costs are :
Cost of purchase, maintenance, installation charges and operational charges

Sunk Costs

- Sunk costs are those costs that are incurred whether there is an expansion or not. These are costs which are made once and cannot be altered, increased or decreased. These types of costs are based on prior commitment; thus cannot be revised or recovered.
- For example: if a firm hire a machine ,it has to bear the rent and other operational charges, which are the sunk cost of the organization.

Short run and Long run

- **Short run**

- a period of time where at least one factor is fixed, usually capital stock is fixed, and all others are variable.

- **Long run**

- a time period where all factors of production, even the capital stock, can be varied

Costs in the Short Run & Long Run

- Short run:
Some inputs are fixed (*e.g.*, factories, land).
The costs of these inputs are *FC*.
- Long run:
All inputs are variable
(*e.g.*, firms can build more factories,
or sell existing ones).
- In the long run, *ATC* at any Q is cost per unit using the most efficient mix of inputs for that Q (*e.g.*, the factory size with the lowest *ATC*).

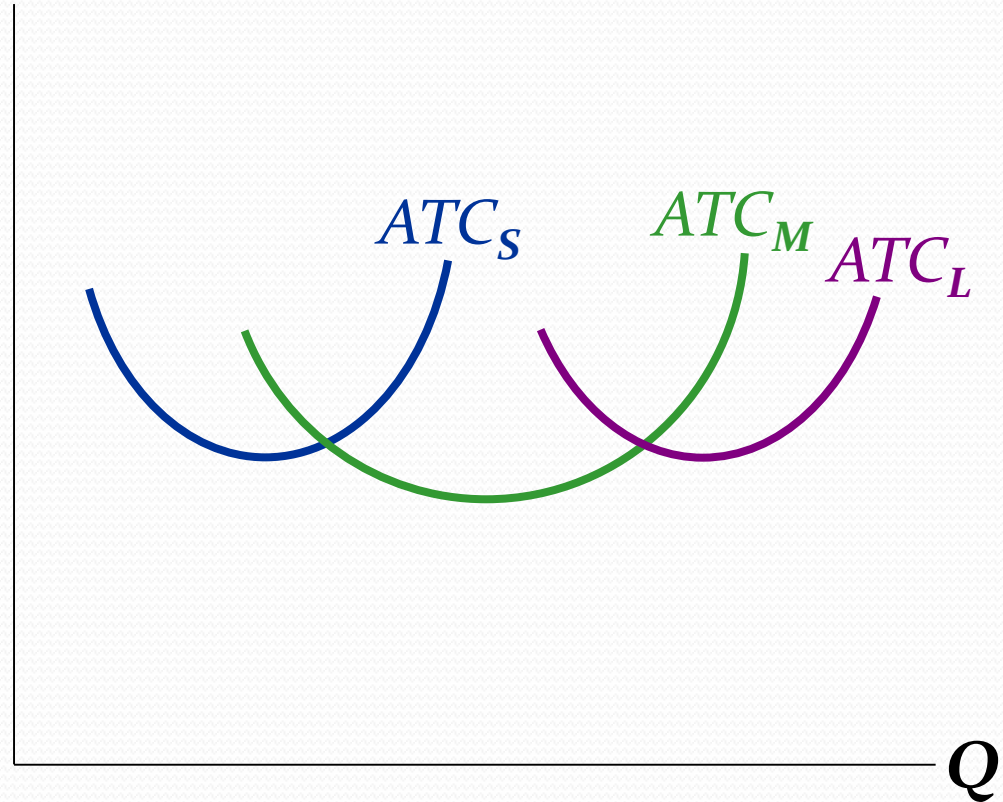
EXAMPLE 3: LRATC with 3 factory Sizes

Firm can choose
from 3 factory sizes: **S, M, L.**

Avg
Total
Cost

Each size has its
own *SRATC* curve.

The firm can
change to a
different factory
size in the long run,
but not in the short
run.

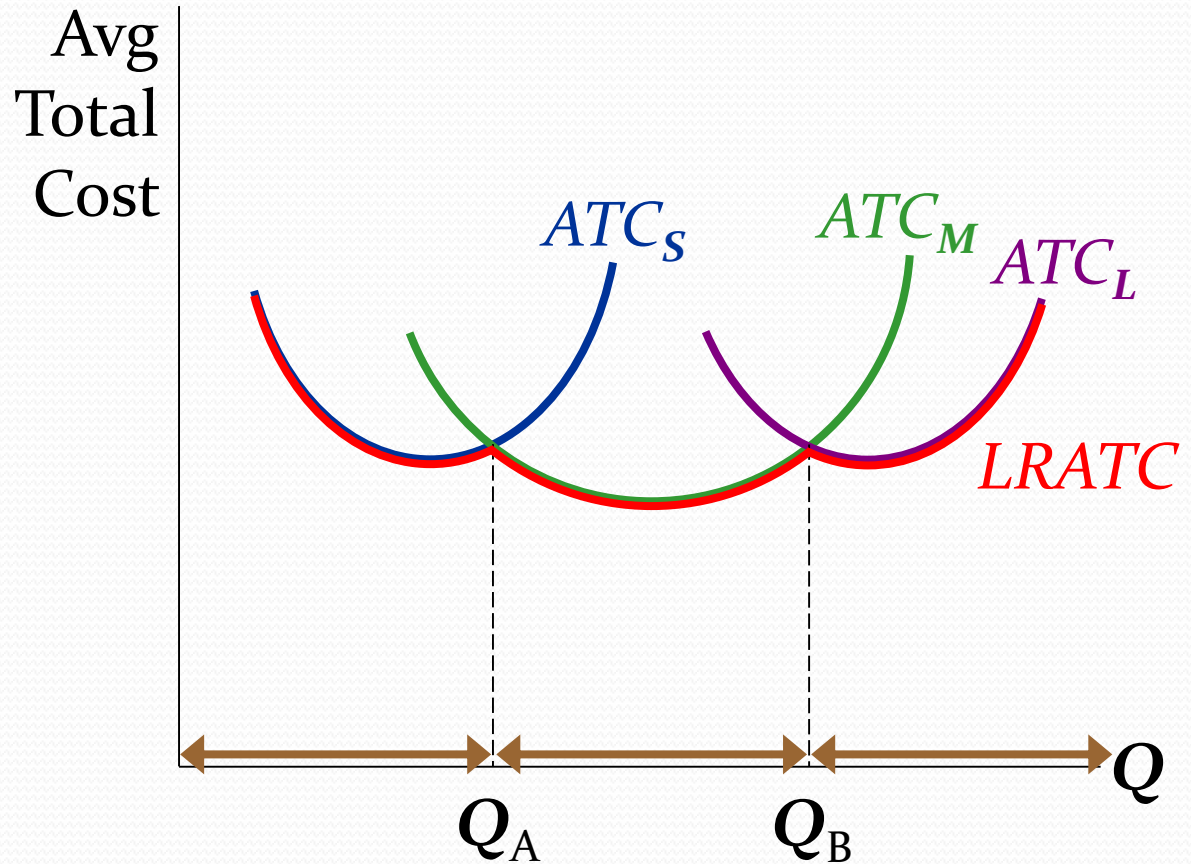


EXAMPLE 3: LRATC with 3 factory Sizes

To produce less than Q_A , firm will choose size **S** in the long run.

To produce between Q_A and Q_B , firm will choose size **M** in the long run.

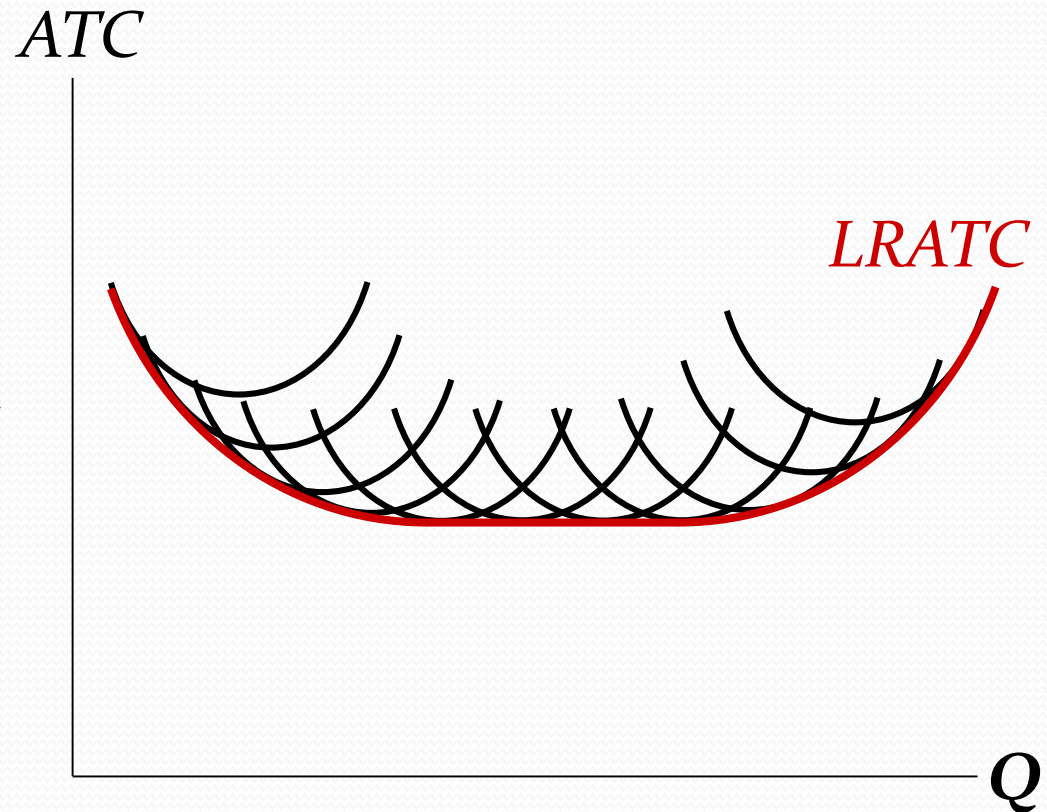
To produce more than Q_B , firm will choose size **L** in the long run.



A Typical LRATC Curve

In the real world, factories come in many sizes, each with its own *SRATC* curve.

So a typical *LRATC* curve looks like this:

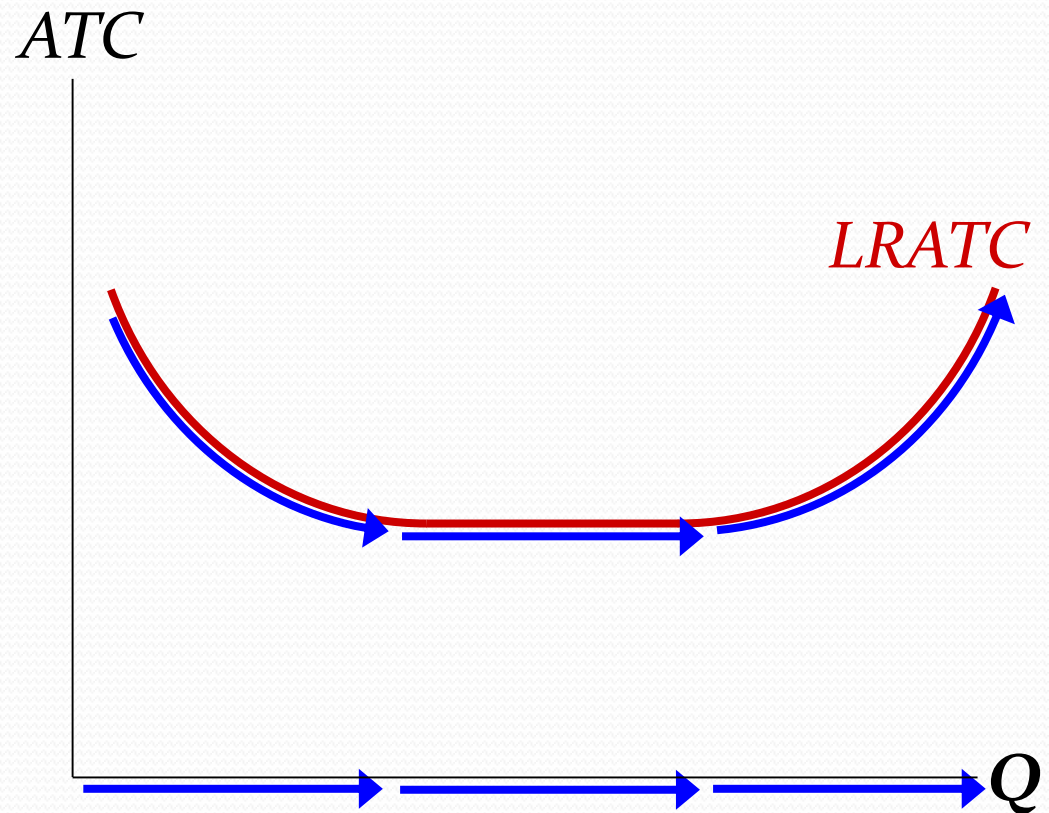


How ATC Changes as the Scale of Production Changes

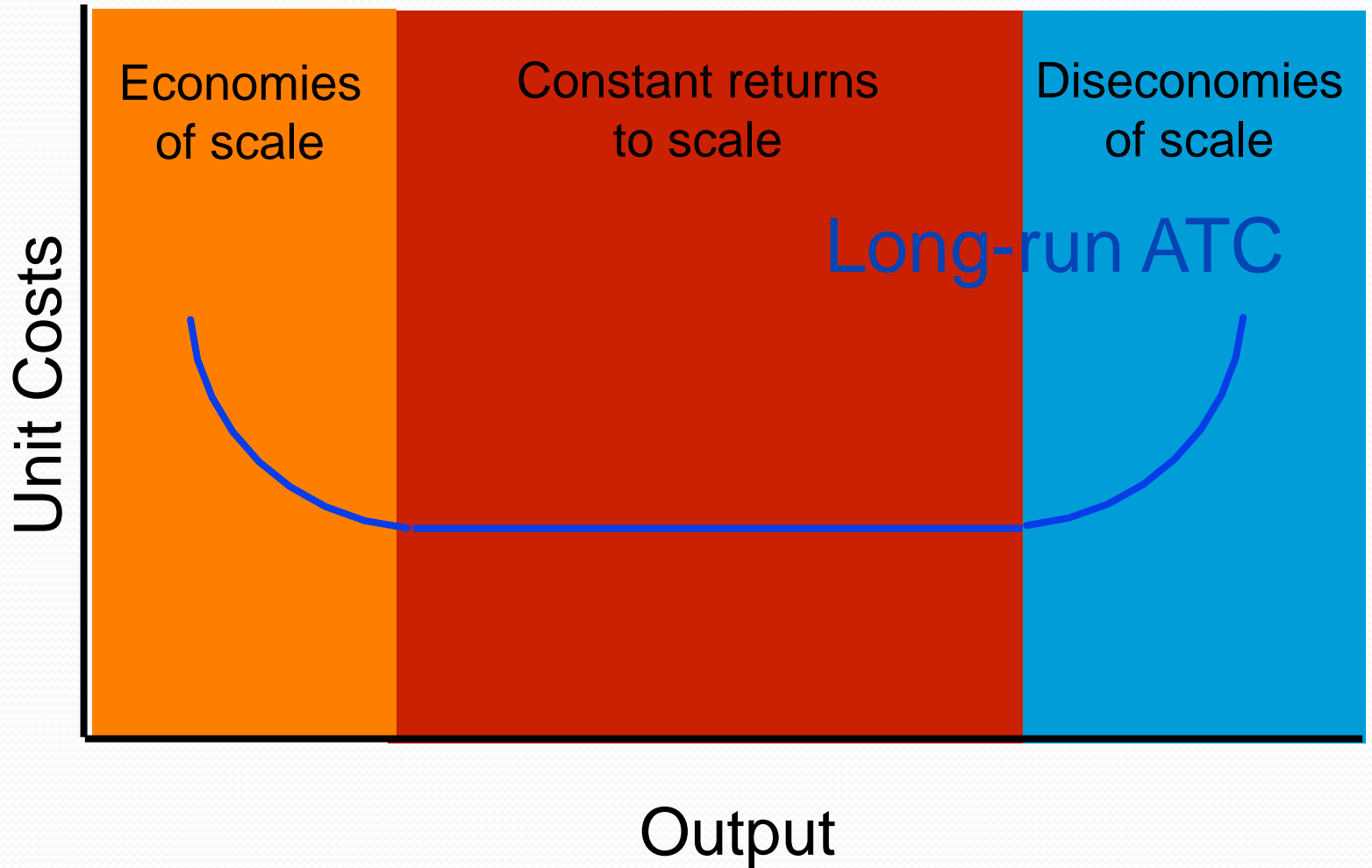
Economies of scale: ATC falls as Q increases.

Constant returns to scale: ATC stays the same as Q increases.

Diseconomies of scale: ATC rises as Q increases.



Long-Run ATC Curves



How ATC Changes as the Scale of Production Changes

- **Economies of scale** occur when increasing production allows greater specialization: workers more efficient when focusing on a narrow task.
 - More common when Q is low.
- **Diseconomies of scale** are due to coordination problems in large organizations.
E.g., management becomes stretched, can't control costs.
 - More common when Q is high.

The cost Function

- The cost of function expresses a functional relationship between total cost and factors that determine it. Usually, the factors that determine total cost of production (C) of a firm are the output(Q), the level of technology (T), the prices of factors (Pf) and the fixed factors (F).

The cost functions becomes :

$$C = f(Q, T, P_f, F)$$

Cost function requires multi-dimensional diagrams which are difficult to draw. So assuming that all other factors are constant.

- Which means that the total cost (C) is a function (f) of output (Q).

$$C = f(Q)$$

Cost function is observed both in short run and long run.

Types of cost functions

- There are namely three types of cost functions :
 1. Linear Cost functions
 2. Quadratic cost functions
 3. Cubic cost functions

Linear Cost functions

- It shows a linear or straight line relationship between the output and cost. The variable cost change in same amount as the output of the organization . It is expressed as :

$$TC = a + bQ$$

TC = Total Cost

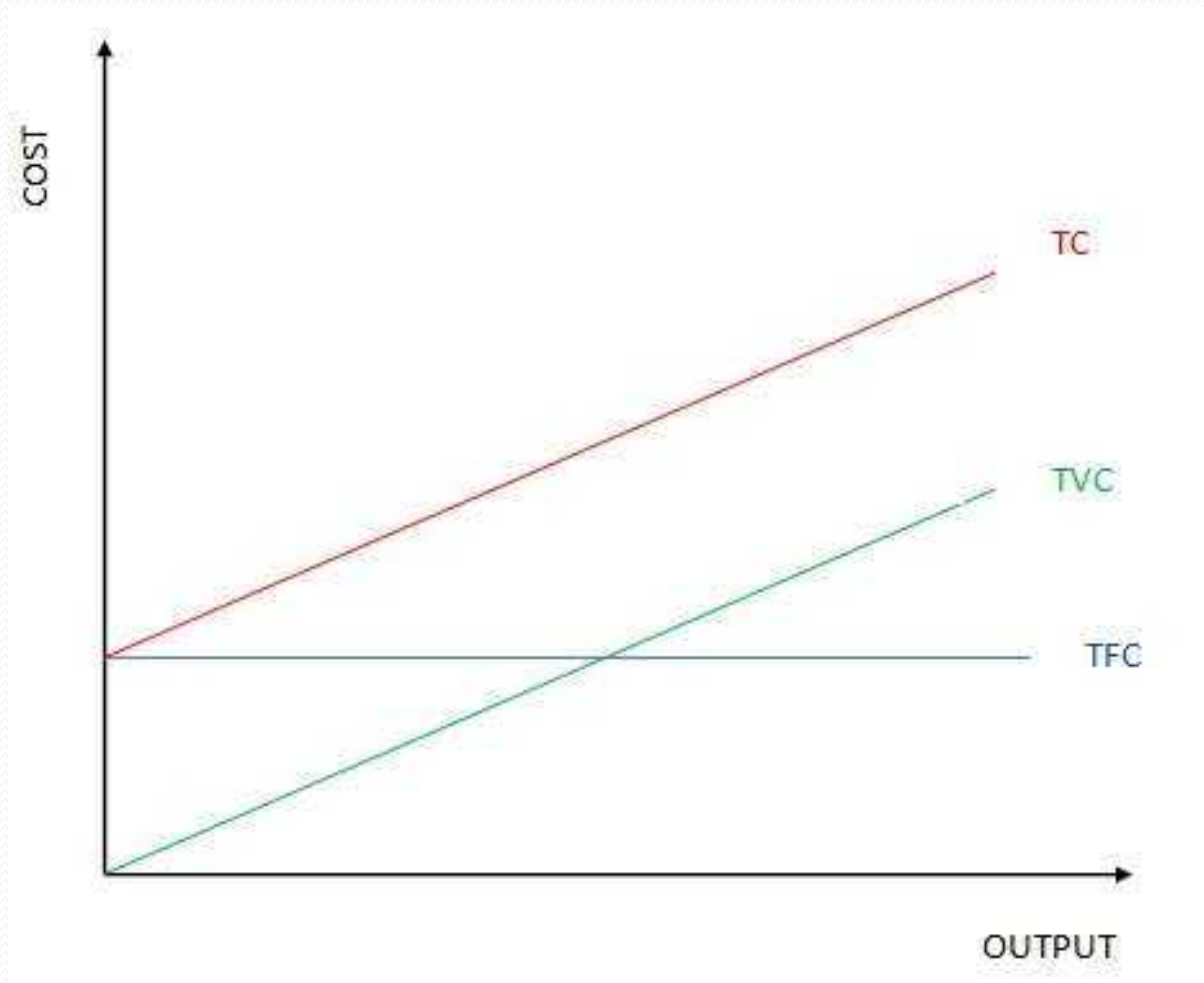
Q= quantity produced

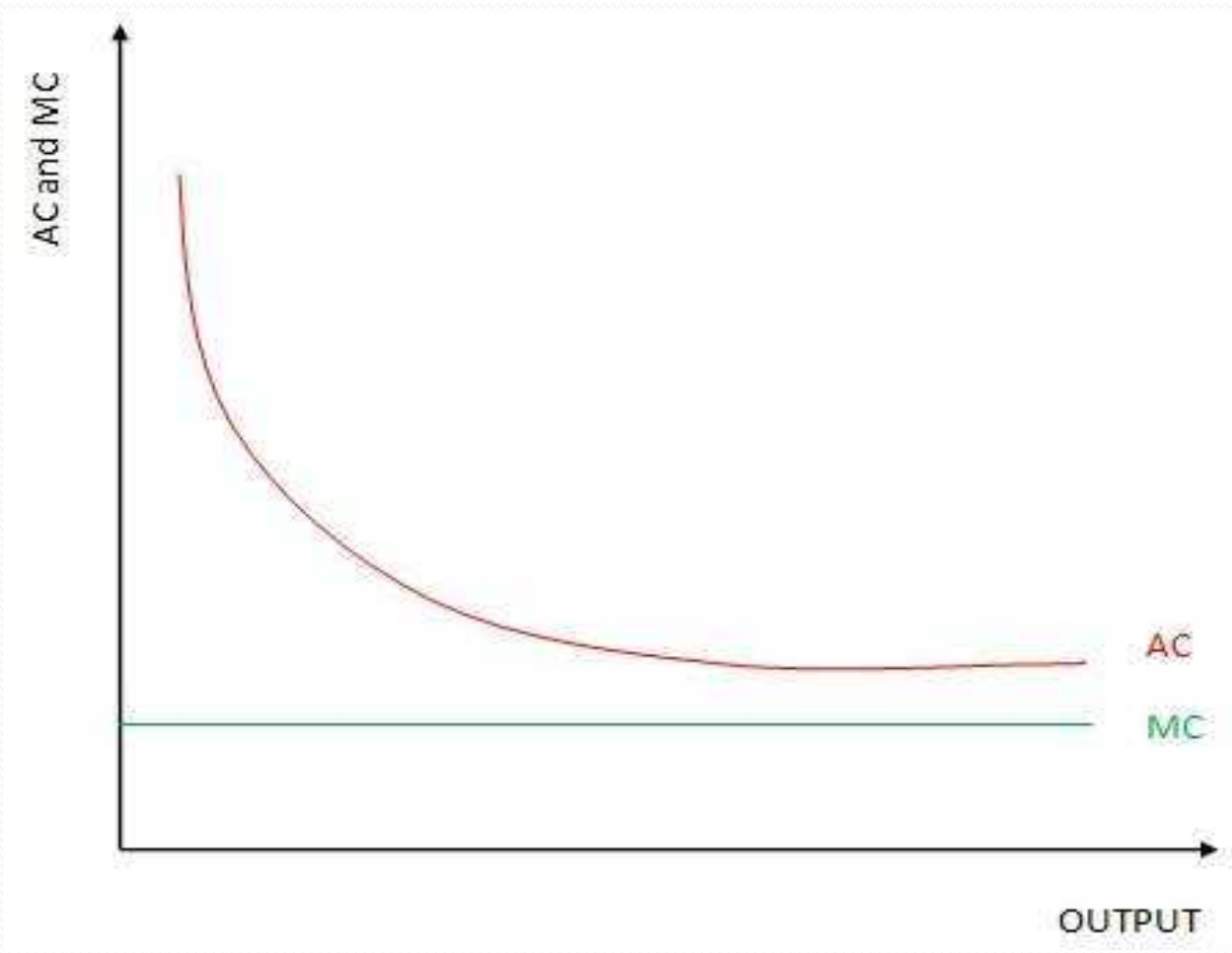
a = is the constant that indicates that value of total cost when output is zero

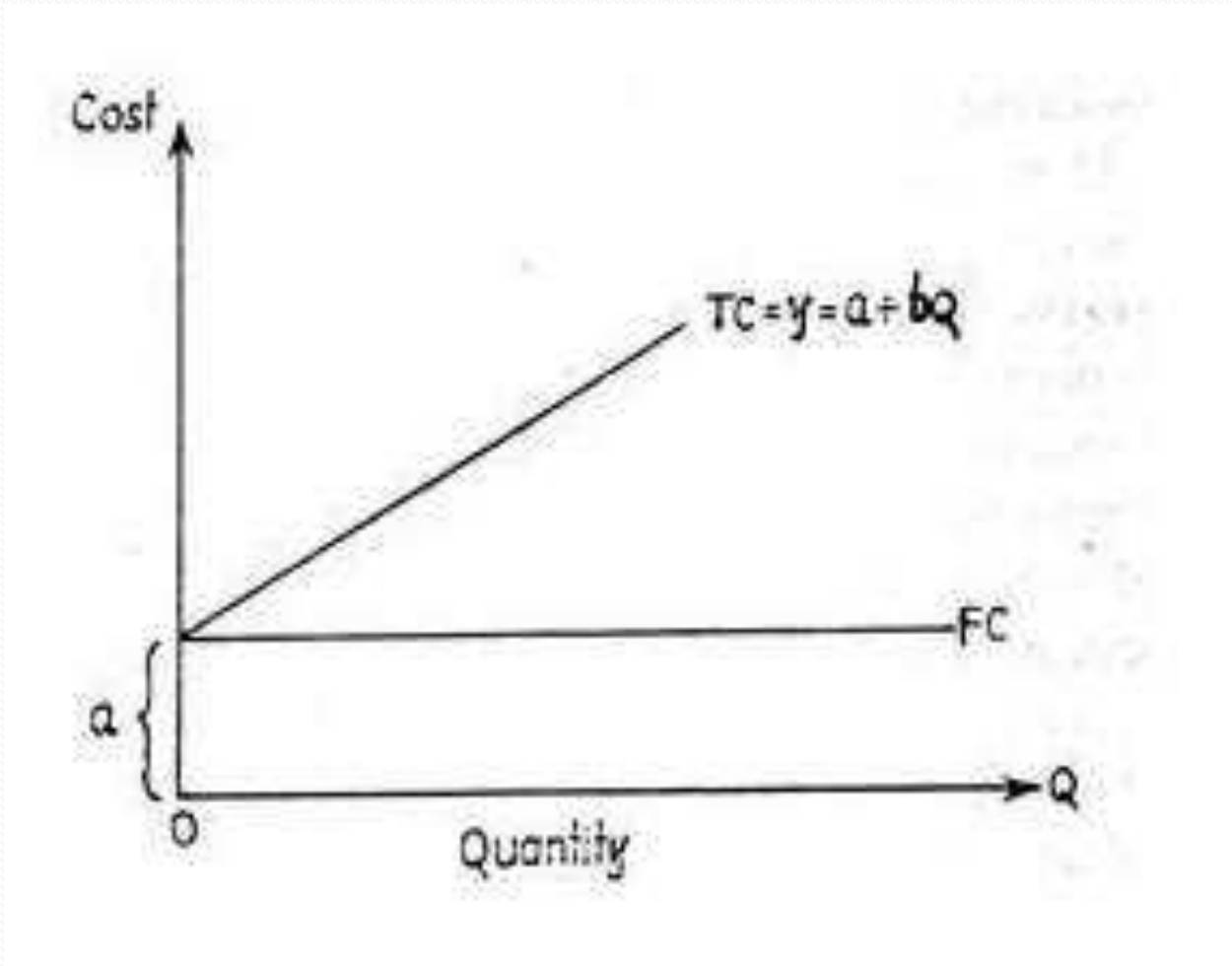
b = slope of the straight line that depicts relationship between cost and output.

| Quantity 'Q' | FC | VC | TC=FC+VC | AFC=FC/Q | AVC=VC/Q | ATC=TC/Q | MC |
|--------------|----|----|----------|----------|----------|----------|----|
| 0 | 10 | 0 | 10 | | | | |
| 1 | 10 | 2 | 12 | 10.0 | 2 | 12.0 | 12 |
| 2 | 10 | 4 | 14 | 5.0 | 2 | 7.0 | 2 |
| 3 | 10 | 6 | 16 | 3.33 | 2 | 5.33 | 2 |
| 4 | 10 | 8 | 18 | 2.5 | 2 | 4.5 | 2 |
| 5 | 10 | 10 | 20 | 2.0 | 2 | 4.0 | 2 |
| 6 | 10 | 12 | 22 | 1.67 | 2 | 3.66 | 2 |
| 7 | 10 | 14 | 24 | 1.43 | 2 | 3.42 | 2 |
| 8 | 10 | 16 | 26 | 1.25 | 2 | 3.25 | 2 |
| 9 | 10 | 18 | 28 | 1.11 | 2 | 3.11 | 2 |
| 10 | 10 | 20 | 30 | 1 | 2 | 3.0 | 2 |

Linear Cost Function Graph







Quadratic cost functions

- If there is diminishing return to the variable factor the cost function becomes quadratic. There is a point beyond which TP is not proportionate. Therefore, the marginal product of the variable factor will diminish.
- And if TP actually falls MP will be negative. In other words, there is a point beyond which additional increases in output cannot be made. So costs rise beyond this point, but output cannot.

- It is expressed :

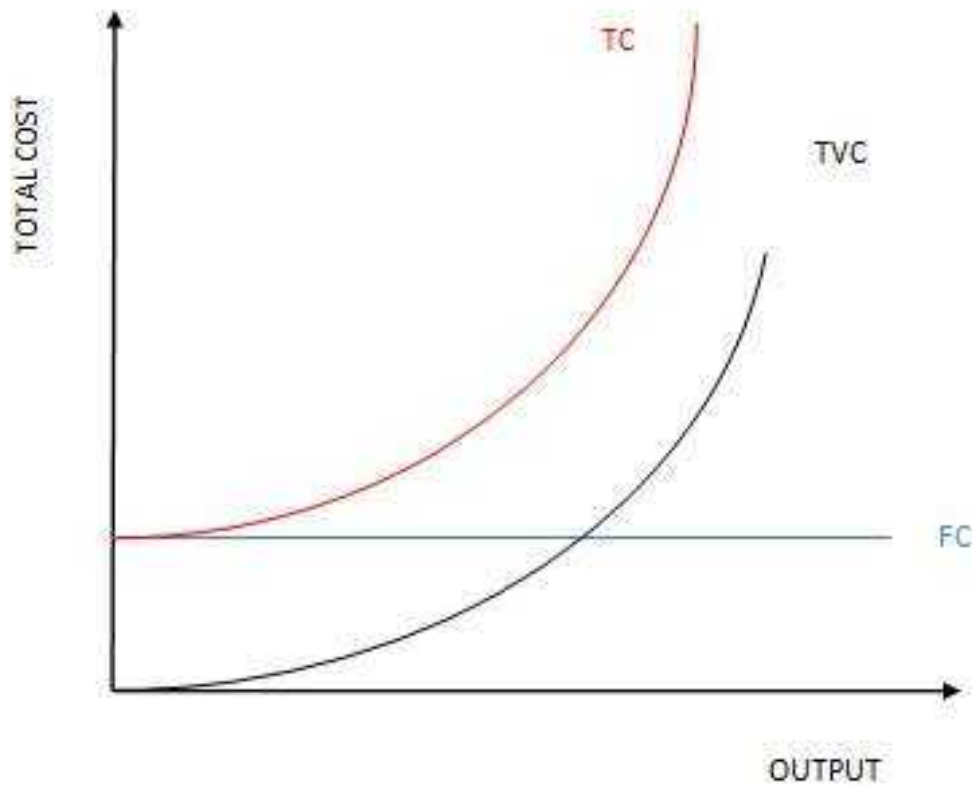
$$TC = a + bQ + cQ^2$$

'a' = is constant indicating the value of total cost when the output of the firm is zero. The value of total cost in such a case will be equal to the fixed cost of the firm as at this point the variable cost of the firm will be zero.

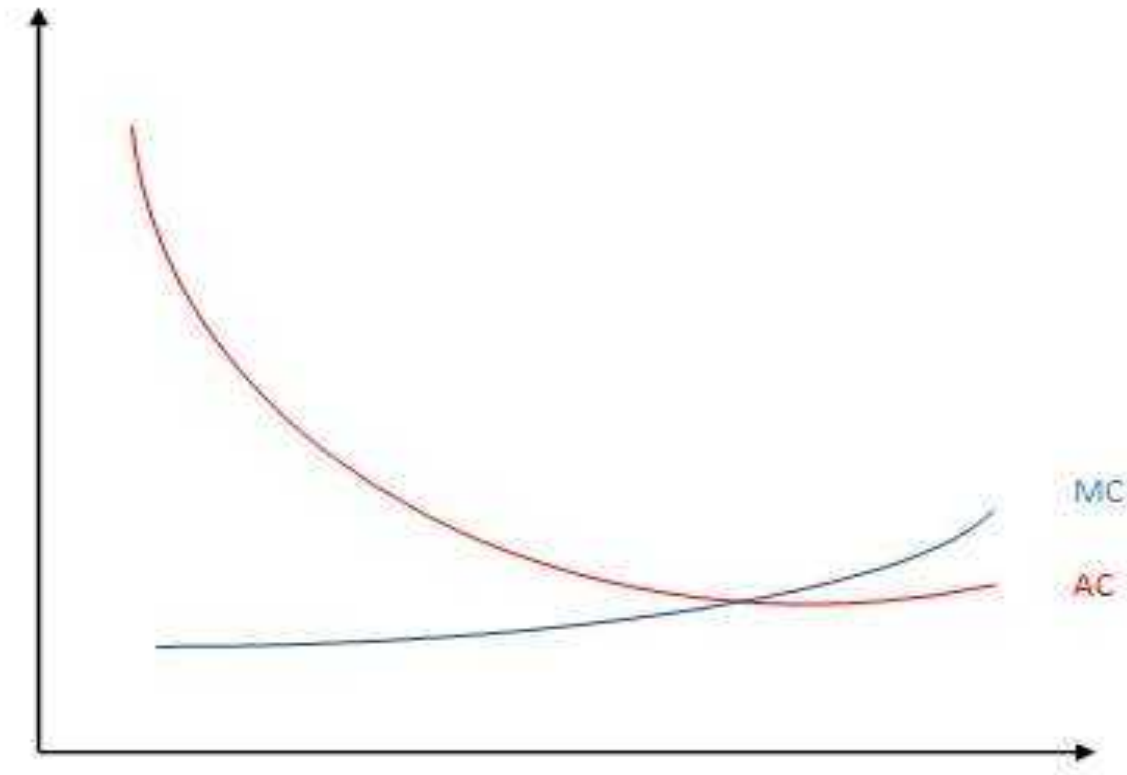
'b' and 'c', = is the constants and indicate the slope of the quadratic cost function

| Quantity 'Q' | FC | VC | TC=FC+VC | AFC=FC/Q | AVC=VC/Q | ATC=TC/Q | MC= |
|--------------|----|------|----------|----------|----------|----------|------|
| 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 10 | 4 | 14 | 10.0 | 4 | 14.0 | 4 |
| 2 | 10 | 10 | 20 | 5.0 | 5.0 | 10.0 | 6 |
| 3 | 10 | 17 | 27 | 3.33 | 5.66 | 9 | 7 |
| 4 | 10 | 26.4 | 36.4 | 2.5 | 6.6 | 9.1 | 9.4 |
| 5 | 10 | 37 | 47 | 2.0 | 7.4 | 9.4 | 10.6 |
| 6 | 10 | 50 | 60 | 1.67 | 8.33 | 10.0 | 13 |
| 7 | 10 | 67 | 77 | 1.43 | 9.57 | 11 | 17 |
| 8 | 10 | 92 | 102 | 1.25 | 11.5 | 12.75 | 25 |
| 9 | 10 | 132 | 142 | 1.11 | 14.66 | 15.77 | 40 |

Quadratic Cost Function Graph

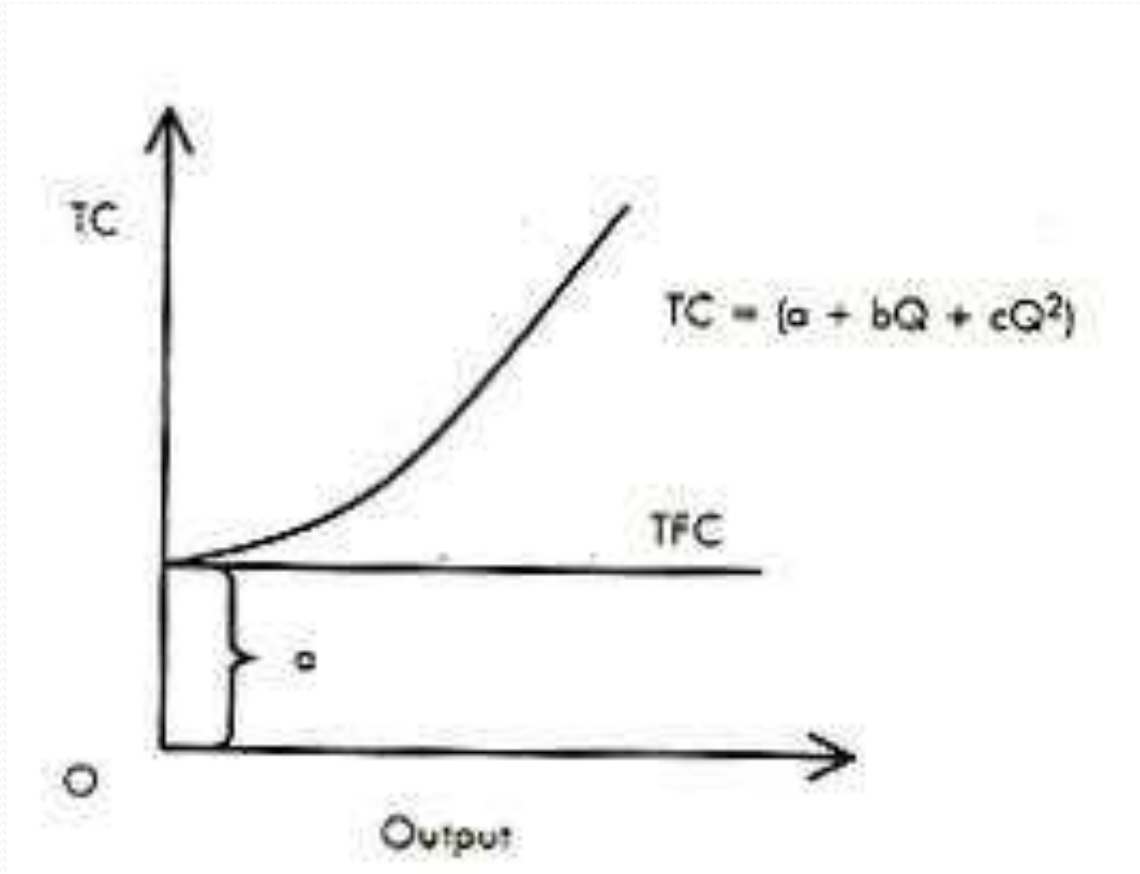


AC and MC



Output

Quadratic Cost Function Graph



Cubic cost functions

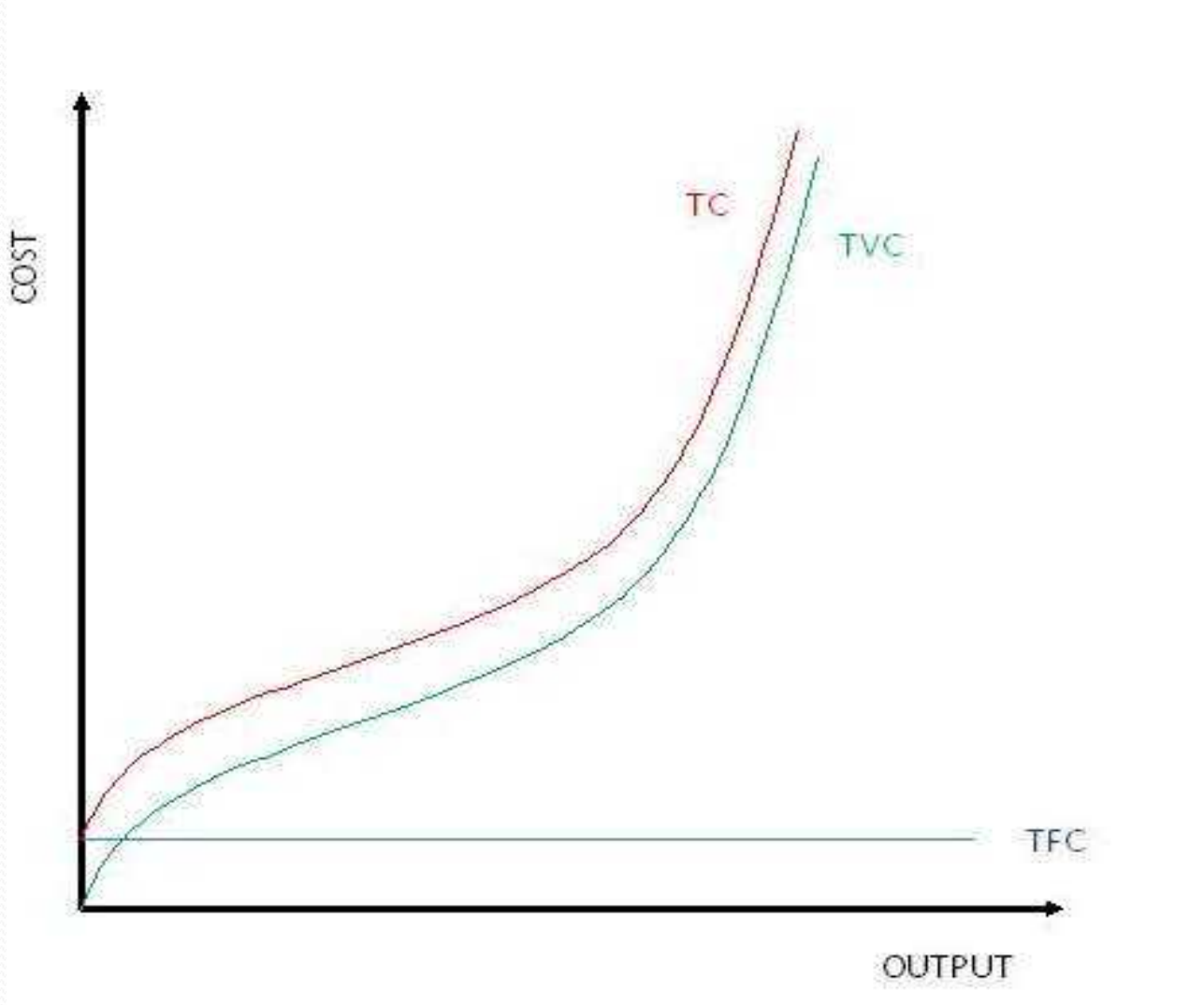
- the shape of the total variable cost curve is like inverse 'S'. This shape indicates that if more and more of the variable factor is applied to the fixed factor then the output of the firm initially increases at an increasing rate then at a constant rate and finally it starts to diminish.
- The Average Variable cost initially decreases, reaches its minimum and finally it starts to increase again. This also results in the increase in the Total Variable Cost and the Total Cost at a diminishing rate initially, then at a constant rate at finally at an increasing rate

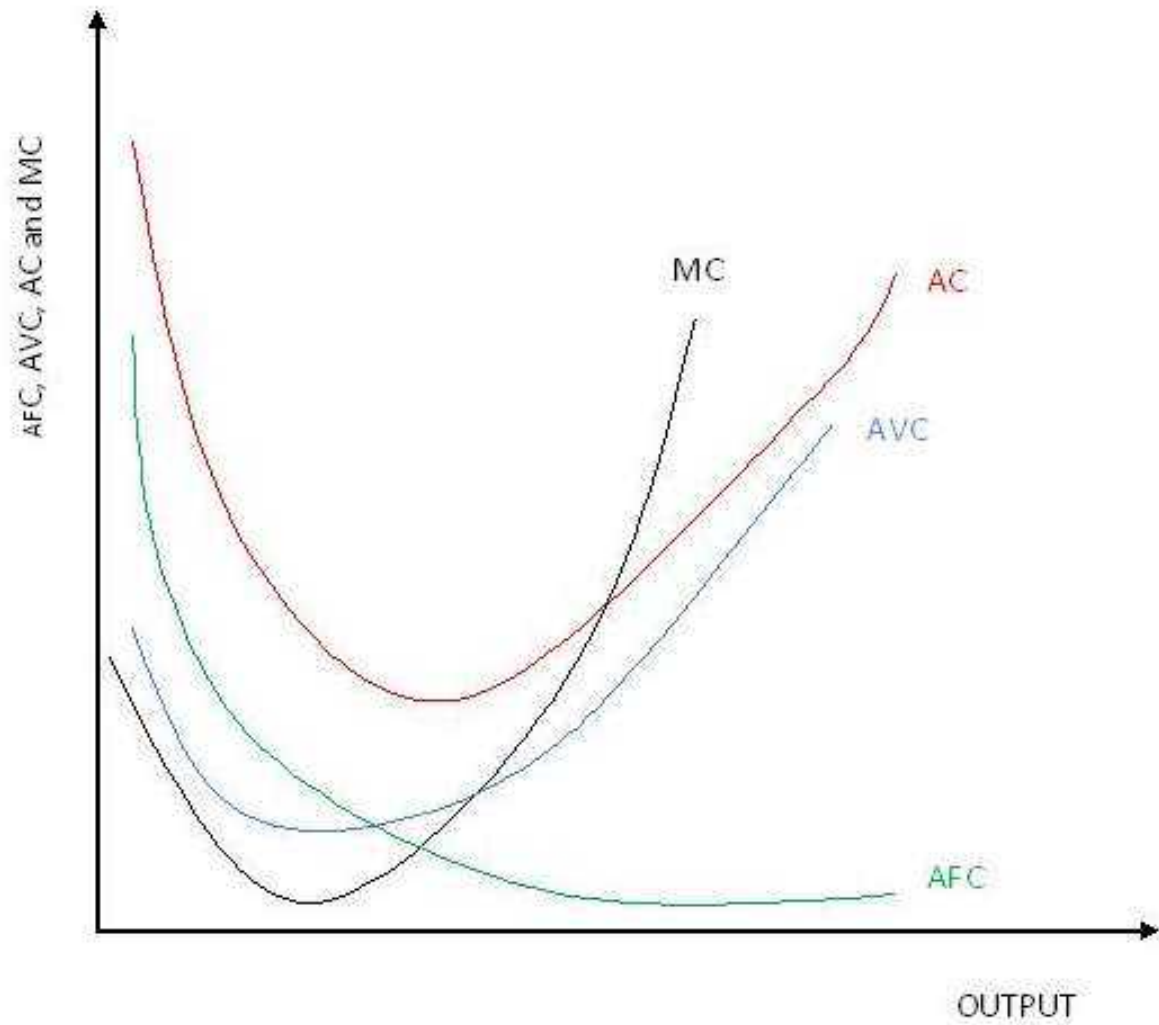
- The Cubic Cost function can be mathematically depicted as follows:

$$TC = a + bQ - cQ^2 + dQ^3$$



| Quantity'Q' | FC | VC | TC=FC+VC | AFC=FC/Q | AVC=VC/Q | ATC=TC/Q | MC |
|-------------|----|----|----------|----------|----------|----------|----|
| 0 | 10 | 0 | 10 | 0 | 0 | 0 | 0 |
| 1 | 10 | 4 | 14 | 10 | 4 | 14 | 4 |
| 2 | 10 | 7 | 17 | 5 | 3.5 | 8.5 | 3 |
| 3 | 10 | 9 | 19 | 3.33 | 3 | 6.33 | 2 |
| 4 | 10 | 10 | 20 | 2.50 | 2.50 | 5 | 1 |
| 5 | 10 | 11 | 21 | 2 | 2.2 | 4.2 | 1 |
| 6 | 10 | 14 | 24 | 1.67 | 2.33 | 4.00 | 3 |
| 7 | 10 | 18 | 28 | 1.43 | 2.57 | 4.00 | 4 |
| 8 | 10 | 24 | 34 | 1.25 | 3.00 | 4.25 | 6 |
| 9 | 10 | 32 | 42 | 1.11 | 3.55 | 4.66 | 8 |
| 10 | 10 | 42 | 52 | 1.00 | 4.2 | 5.2 | 10 |





For example

- There is firm producing footballs. It incurs a fixed cost of 2000\$. The cost function of firm is given as :

$$TC = 2000 + 15Q - 6Q^2 + Q^3$$

Find out the following from the given cost functions:

- (a) What is the fixed cost when quantity is 2000 units.
- (b) What is AFC when quantity is 2000 units?
- (c) What is TVC when quantity is 20 units?
- (d) What is ATC when quantity is 20 units?
- (e) What is MC when quantity is 20 units?

(a) What is the fixed cost when quantity is 2000 units.

TFC remain constant. $TFC = 2000\$$

(b) What is AFC when quantity is 2000 units?

$$AFC = TFC / Q = 2000 / 2000 = 1 \$$$

(c) What is TVC when quantity is 20 units?

$$TC = 2000 + 15Q - 6Q^2 + Q^3$$

$$TVC = 15Q - 6Q^2 + Q^3$$

$$TVC = 15(20) - 6(20)^2 + (20)^3$$

$$TVC = 5900 \$$$

(d) What is ATC when quantity is 200 units?

$$ATC = TC / Q$$

$$TC = 2000 + 15Q - 6Q^2 + Q^3$$

$$TC = 2000 + 15(20) - 6(20)^2 + (20)^3$$

$$TC = 2000 + 5900$$

$$TC = 7900 \$$$

$$ATC = 7900 / 20 = 395\$$$

(e) What is MC when quantity is 20 units?

$$MC = \Delta TC / \Delta Q$$

$$TC = 2000 + 15Q - 6Q^2 + Q^3$$

$$MC = 15 - 12Q + 3Q^2$$

$$MC = 15 - 12(20) + 3(20)^2$$

$$MC = 975 \$$$

Exercise

Q.1 By using following equation calculate a total Fixed Cost, Average Fixed Cost, Total Variable Cost, Average Variable Cost , Average Total Cost and Marginal Cost when quantity is 10 units.

$$TC = 4000 + 5Q + 10Q^2$$